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June 10, 2024

Review of the doctoral dissertation of **Meizhi Qian**, MSc,
employed at East China Normal University, entitled

*Topological derivative method in control and design
of distributed parameter systems on metric graphs*

prepared on order from Systems Research Institute of the Polish Academy
of Sciences from April 5, 2024.

General description. The dissertation starts with the Introduction, aimed at introducing the general philosophy of shape and topological optimization. Some space is devoted here to continuous problems in $\mathbb{R}^2$ and $\mathbb{R}^3$, not related to the rest of the work, but it may be justified, because in such a setting it is easier to describe an approach based on the Steklov–Poincaré operator. This operator is later used for networks of pipes or beams. The introduction should probably contain also a description of the turnpike property, which is used extensively in further chapters, but never explicitly defined.

Chapter 2 contains the summary of Timoshenko beam modelling, very terse, but possible to understand for readers having the previous knowledge of the subject. As I understand, it's goal is mainly to introduce the type of equations which are considered later.

Chapter 3 deals with control problem for a single beam or a similar system of equations defined on an interval. It contains first order necessary optimality conditions for the static case, the second order sufficient condition of optimality and some numerical examples.

Chapter 4 introduces the network of beams and defines the domain decomposition technique for computation of topological derivatives. The topological variation of the network consists in replacing the single node at the fork of three beams by a small cycle. Then the neighbourhood of the cycle is replaced by the exact analytical solution. Again the necessary and sufficient conditions for the optimal solution (using the exact formulas mentioned above) are obtained. Some numerical examples follow.

Chapter 5 concerns a little different problems. In previous chapters the position of the point along the length of the beam was treated as an analogue of time, thus allowing the application of the classical control theory. Here the time dependent hyperbolic equations on variable length one-dimensional arcs are considered. The analysis is performed by using the spectral decomposition method. The main results of this chapter encompass the turnpike property for basis functions in this decomposition and for the whole optimality system. Finally the topological derivative for the optimal network in terms of the small parameter (the size of the cycle) is computed for the static network containing single fork and scalar static equations on branches.

Main achievements. The subject of the dissertation is well selected in view of its applicability to real world problems and the possibility of obtaining new, interesting results. The approach to the static networks via control theory, while not exactly new, shows here its flexibility. I would enumerate the principal original findings as follows.

1. In Chapter 3 it is proved that the sufficient optimality conditions for a single beam imply coercivity for the tracking type of problems, thus ensuring the existence of optimal solution after the finite element approximation. The numerical procedure used here is also interesting.
2. In Chapter 4 the computation of the Steklov-Poincaré operator for the fork with a small cycle required advanced technical abilities and may constitute the pattern for more complicated cases. Observe that the configuration considered has only three branches and equal lengths of cycle arcs.
3. The main result of the dissertation is contained, in my opinion, in Chapter 5. The theorems on turn-pike properties seem original. Also the technique using the spectral decomposition in complex domain required a lot of technical skills. Finally, the implicit formula for the topological derivative for static fork with cycle is worth of mention.

In general, undertaking the project requiring the notoriously complicated analytical computations of classical control theory required a lot of daring. The result obtained may lead to further development of the theory of structural networks. The bibliography is sufficient and reflects the current research.

Remarks. I am not a native speaker, so I will not comment on the language. As concerns formulas and derivations, I haven't notice errors or misprints, but the text is very dense, so some could slip through. The main substantive remarks are listed below.

1. Introduction. The introduction of distributed systems here should be more clearly justified, since the rest of the work deals with networks.
2. Introduction. I would give here one of the definitions of the turn-pike property. In further chapters it is proved (exponential version), but never clearly defined.
3. Chapter 2. The treatment of joints in the beams theory is a little simplistic. It models only the case of equilibrium of forces and moments together with continuity condition. This approximates rigid joints, but there are many other types. Some comment would be in place.
4. Chapter 3. It should be more clearly stressed, that further results are obtained only for tracking type of goal functionals. Is this condition essential ?
5. Chapter 3. It would be helpful to see equations for example 2 in an explicit form, maybe for initially straight beam with constant circular cross-section.
6. Chapter 3. It should be mentioned, that optimal control problems could be approached also in another way, without using finite elements. But, anyway, the theoretical analysis guarantees the existence of solutions, so has an intrinsic valuable.
7. Chapter 4. As mentioned already, the derivations in this chapter show technical skills and deep understanding of the control theory as well as particular problems under consideration. Again, the finite elements approach is not the only one, but possibly gives more insight?
8. Chapter 5, page 87. Here the definition of turn-pike property would come handy, the reader may not immediately know, what this inequality means (see also eq. 5.42), and, as stressed before, this is one of the main results of the dissertation.
9. Chapter 5. Theorem 10 constitutes, i think, main result of the dissertation. The reader would appreciate more detailed comments about its value and applications (e.g. in numerics).
10. Chapter 5, page 119. The examples are very interesting, but would gain with some follow through. Maybe one should write down the exact form of the topological derivative of $J(u)$ in terms of φ , p and Λ_ε . This would connect the examples with Introduction.

11. General remark. In the text of the Dissertation more stress should be given to non-structural interpretations of the models, for example as flow in pipes. In such cases the Kirkhoff-type conditions at joints make more immediate sense.

Conclusions. In view of the above comments and remarks I hereby state, that the Dissertation of **Meizhi Qian** satisfies all the requirements of appropriate enactments for obtaining Ph.D. degree (art. 187 Ustawy z dnia 20 lipca 2018 r. - Prawo o szkolnictwie wyższym i nauce (Dz.U. 2018 poz. 1668 z óżn. zm.)). I recommend acceptance of the Dissertation for public defence.

In addition I propose to award commendation to the doctoral candidate and her thesis. It is justified by the substantial technical skills shown in carrying out an important and highly non-trivial research program. It should be also taken into account, that the candidate has already substantial publication record.

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A handwritten signature in blue ink, appearing to read 'Żochowski', is written below the printed name.