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Referee's Report on the Doctor's Thesis:

Meizhi Qian, Topological Derivative Method in Control and Design of Distributed Parameter Systems on Metric Graphs

presented to the Department of Modeling and Optimization of Dynamic Systems
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The field of science: Engineering technology
discipline: automation, electronics, electrical engineering and space technologies

prepared according to the letter of April 24, 2024 from the vice-head of the Systems
Research Institute, dr hab. inż. Jan. W. Owsinski.

The references to the items of the list of References of the Thesis reviewed will be
denoted by [n], while the new references introduced in this review will be denoted
by [Rn]. Equations in the present review will be named (Rn).

1. The résumé of the Doctorand (the Candidate): Meizhi Qian

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Sept. 2015 – June. 2019 Zhejiang Sci-Tech University – Hangzhou,
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The scientific outcome:

1. Meizhi Qian and Shengfeng Zhu, A level set method for Laplacian eigenvalue optimization subject to geometric constraints, *Computational Optimization and Applications*, 82 (2022) 499-524.
2. Meizhi Qian, Xindi Hu, and Shengfeng Zhu, A phase field method based on multi-level correction for eigenvalue topology optimization, *Computer Methods in Applied Mechanics and Engineering* 401 (2022) 115646.
3. Xindi, Hu, Meizhi Qian, and Shengfeng Zhu, Accelerating a phase-field method by linearization for eigenfrequency topology optimization. *Structural and Multidisciplinary Optimization*, 66 (2023), 242.
4. Martin Gugat, Meizhi Qian, and Jan Sokołowski, Topological derivative method for control of wave equation on networks, 2023, 27th International Conference on Methods and Models in Automation and Robotics (MMAR), Międzyzdroje, Poland, 2023
5. Martin Gugat, Meizhi Qian, and Jan Sokołowski. Network design and control: Shape and topology optimization for the turnpike property for wave equation, (2023). (submitted)
6. Nikolai P. Osmolovskii, Meizhi Qian, and Jan Sokołowski. Net-work optimality conditions. *Control and Cybernetics*, (2024) (accepted).

Thus, the hitherto prepared papers concerned the eigenvalue optimization problems appearing in : Shape Optimization, Finite Element Analysis, Optimal Control and the Topological Derivative Method. The Candidate shows high skills in programming in the languages: C, C++, Python and in a creative usage of Matlab, and Freefem++ developed by F. Hecht.

2. Organization of the Thesis and description of its subject

The Thesis is composed of independent articles concerning different subjects.

The Introduction presents a brief overview of the problems discussed and the methods used.

Chapter 2 concerns theory of spatial elastic deformations of vibrating bars, initially curved developed by Antman, Simo, Rodriguez, Leugering, Strohmeier et al.

Sec.2.1.1 refers to a geometrically exact beam model (GEM), while Sec 2.1.2 concerns its reformulation called IGEM. The presentation is extremely compact aimed at reducing the whole theory to an abstract system (2.5). Both GEM and IGEM neglect the transverse shear deformations. Sections 2.2 and 2.3 refer to the linearized theory of Timoshenko beams in which these effects are modelled in the simplest possible way.

Chapter 3 concerns selected control problems of networks composed of linearly elastic bars.

Sec.3.1 recalls the formulation and the methods of solving the optimal control problem expressed by Eqs.(3.1). To approach the problem effectively one should introduce Pontriagin's function and then solve the LMP problem (local minimum principle formulation), page 22. The sufficient optimality conditions are recalled in Sec 3.1.2. The positive definiteness of the crucial quadratic form is linked with solvability of the Riccati equation, see Sec 3.1.3.

In Sec.3.2 a model control problem of minimization of the functional (3.8), with the integrand chosen later in the form (3.21), on the interval $[0, l]$ is considered, l being one of the unknowns subject to a box condition (3.9); the additional condition $(3.7)_3$ is expressed by an affine function of the end values of the scalar control function $z(x)$. The problem is eventually expressed in the form of the conditions set up in Theorem 5. The equations involve a new unknown called the adjoint variable $p(x)$. The sufficient conditions for the triple: $(l, z(\cdot), f(\cdot))$ to be a weak local minimum in problem (3.7)-(3.9) are formulated and proved, see Theorem 6. Theorem 8 clears up the question of positive definiteness of the form Q on a specific subspace. The theoretical results of Secs 3.1 and 3.2 are illustrated by numerical examples in Secs 3.3.1 and 3.3.2. In Example 1 (Sec.3.3.1) the unknown is a scalar function $z(x)$. In Example 2 (Sec.3.3.2) the unknown is a vector function of 6 components.

Chapter 4 concerns a specific class of optimum design problems of frames composed of Timoshenko bars. The formulation is partly based on the Doctor's thesis by E. Ogiermann [61] in which sensitivity of the compliance of frames with respect to replacing the "star connection" of the bars coming to a given joint by intermediate connections, called there "forming a hole", has been computed and treated as an optimization tool for designing frames. The frames are considered as obeying linear laws of mechanics. Ogiermann derived explicit asymptotic formulae for the topological derivatives of the compliance of 3D and 2D frames. The Candidate considered a more difficult problem of deriving the topological derivative of the cost functional given by (4.33); the Ogiermann results on the compliance minimization problem are here not very helpful. The final results are not fully explicit, see Sec.4.9. Instead the Candidate has shown the numerical results based on the numerical codes; for a sufficiently small parameter ε introduction of a "hole" improves the design, see Fig.4.9. Further parts of the Thesis do not deal with frames made of Timoshenko bars.

Chapter 5 concerns scalar problems on networks, which can be probably be interpreted as propagation of thermal longitudinal waves with the temperature as the only scalar unknown. The Lagrangian of the problem has the form given below Remark 5. , Sec.5.1. The optimality system is formulated in Lemma 1 of Sec.5.1. Its solution is constructed in Sec 5.3 with using the separation of variables involving the eigenfunctions of the eigenvalue problem formulated above Eq. (51.8). In Sec.5.4 a theorem is formulated (Theorem 10, Sec.5.4.2) and proved that the difference between the static and dynamic optimal adjoint functions satisfies a turnpike property (5.45) in which the constant D does not depend on T . In particular the asymptotic property (5.45) has been proved. The turnpike property is eventually illustrated in the numerical examples in Sec.5.5.

The topological derivative for the tree-type network, with respect to appearing of the intermediate triangle as in Fig.5.12, has been constructed explicitly in Sec.5.6. The crucial matrix has the form (5.69). For the multiple cycles this matrix assumes the form shown below Fig.5.15.

The Thesis includes 4-part Appendix A in which the codes in MATLAB are listed. The code in A.1 refers to Sec.3.3.2, the code in A.2 concerns Timoshenko bars in 2D and 3D; the code in A.3 refers to Sec.5.5.1 (a dynamic scalar problem), Sec.A.4 delivers the formula for the derivative of a convolution.

The list of 83 bibliographical items forms the Literature section.

3. Remarks on the approach, the methods and the results

The main assumption of all 1D theories of bars (or rods) is neglecting transverse distortions of the cross section. This assumption is tacitly adopted in the Thesis. This assumption implies that the cross section of a bar is only subject to a rigid body motion in the transverse plane: translations in two directions and rotation around the bar's axis.

All 1D theories of bars take into account the axial deformation and bending deformations in two mutually orthogonal planes. If necessary the assumption of inextensionality may be imposed, which means imposing certain constraints.

The setting of any 1D linear theory of rods depends heavily on whether we omit or not:

- transverse shear deformations
- warping effect due to torsion
- warping effects due to transverse shear in two mutually orthogonal planes.

The Bernoulli-Euler theory neglects all these effects.

The Timoshenko theory takes into account transverse shear deformations.

The Vlasov theory of bars of thin-walled profiles takes into account warping effects due to torsion. The Vlasov-like model extends this approach towards bars of arbitrary cross sections, see [R2].

There exist more complicated theories including some or all of these effects mentioned, e.g. the theory by Librescu and Song taking into account transverse shear deformations as well as warping effects due to torsion and neglecting warping due to shear, see [R2].

In the present paper all warping effects are neglected, keeping no bound on the magnitude of the rotations of the cross sections. Let us stress that neglecting warping due to torsion is justified only in the analysis of bars of circular and annular cross sections, i.e. circular tubes or bars of circular cross sections. Then torsion is not accompanied with warping and all the cross section remain plane upon deformation. In general, the warping occurs, but in case of small deformations of bars of compact cross sections, i.e. of shapes somehow close to a circle, the warping effect is usually neglected. In case of large deformations keeping the plane section assumptions seems controvertible. Only in the case of annular and circular cross sections the warping due to torsion may be omitted.

In the Thesis the Timoshenko bar model is frequently used with the simplest form of the constitutive equations: the bending moments are proportional to the relevant bending measure (given by the axial speed of change of the angle of rotation of the cross section) and the transverse forces are proportional to the relevant measures of shear with the same shear stiffness in both planes, cf the matrix C_N page 17. However, in general, the lower 2 by 2 submatrix of the matrix C_N has non-zero off-diagonal components, see Eqs.(5.17.4,5) and (13.2) in [R2]. Even if the off-diagonal components are neglected, the shear stiffnesses in y and z directions are, in general, different, since only in very special cases (like a rectangular cross section) the effective areas A_z and A_y , given by (4.15) in [R2], are equal.

The characteristic feature of the Timoshenko bar is the lack of coincidence between the axes of principal bending and the axes of principal transverse shear. Consequently, in general, the centroid lies in a different point than the torsion/shear center, which introduces essential difficulties in the analysis of deformation of the joints, since the axes joining centroids do not coincide with the axes joining the centers of torsion/shear. In the Thesis the displacements of the joints determine the displacements of the adjacent bars and the same assumption concerns angles of rotation. But displacements are referred to the axes linking torsion/shear centers while rotations are linked with the lines linking centroids. Hence the continuity conditions at joints are far from being obvious. These problems are omitted in the Thesis, although large rotations are taken into account. This discrepancy between taking into account nonlinear deformations with keeping the simplest constitutive equations along with the simplest description of transverse shear and warping is a feature of many papers on the subject along with the Thesis reviewed. An analysis of accuracy and range of applicability of all these approaches is pending and probably will be cleared up in the future.

Let us express it once again: while analyzing statics of a single Timoshenko bar is an elementary problem, analyzing frames, especially spatial frames is difficult, since displacements are linked with shear centers and angles of rotation are linked with centroids. The interpretation problems appear at joints, where a simple continuity does not take place even if the joints are viewed as being rigid, as usual. Just here a huge difference between the Bernoulli-Euler and Timoshenko models occurs.

Let us note also that the description of warping due to shear given in Ref [R2] is much less complex than that of Gruttmann and Wagner, see the papers cited in Refs[81, R2]. Gruttmann and Wagner take into account so called Poisson ratio effect; then consequently, the centers of shear and the centers of torsion turn out to be different points and the problem of statics becomes essentially difficult, especially the problem of frames composed of bars connected with rigid joints turns out to be an open problem. This discloses that without any passage to large rotations or large displacements, in case of a linear statics framework, we come across problems that cannot be neglected and cannot be ignored. This clears up why civil engineers are not very keen to deal with nonlinear mechanics of skeletal structures.

The problem of non-coinciding of centroids and centers of torsion disappears if the cross sections are bisymmetric, because then centroids and shear centers coincide. Let us repeat, however, that only in case of tubes and rectangular cross sections the structure of the matrix C_N (Table 2.1 and equations in page 17) is correctly assumed.

The explicit solutions to the Timoshenko equations concerning linear statics (Sec.4.6) in case of the absence of the distributed loads can be found in the Thesis by Ogiermann [61] and this source has been used by the Candidate. These solutions have been reported in the literature concerning elastic frames and are routinely programmed within the finite element software, see [R5-R9], also in 3D case involving torsion, see [R7]. The author of the present review has derived them in the planar case for the non-prismatic bars (i.e with varying thicknesses) in his Doctor's Thesis in 1983, see [R9]. The equations linking the end forces/moments with the end displacements/angles of rotations are called the slope-deflection equations in the planar case and the transformation equations of the displacement method in a general case involving torsion. In the Thesis these equations are called Dirichlet-to-Neumann operators, see Sec.4.7. Appropriate elimination of variables lead to the formula called Steklov-Poincaré operator, see Sec.4.8, which leads to the topological derivative result, see Sec.4.9. Figure 4.9 confirms that applying the hole of a certain dimension improves the solution.

Let us note here that within the *linearized approach* the system of equilibrium equations of frames can be reduced to the linear algebraic equations involving the finite number of unknowns; the primal unknown is the vector usually named $\mathbf{q}$ composed of displacements and angles of rotation of joints and selected nodes (if necessary). The three vectors of unknowns are introduced: the mentioned vector $\mathbf{q}$, the vector of discretized strains $\boldsymbol{\varepsilon}$ and the vector $\boldsymbol{\sigma}$ of internal forces (defined as certain integrals of the stress resultants along the bars). The system of equations is formed by:

- the equilibrium equations linking $\boldsymbol{\sigma}$ with the vector $\mathbf{Q}$ of effective loads (R1)
see (7.3) in [R10]
- the constitutive equations linking $\boldsymbol{\sigma}$ with $\boldsymbol{\varepsilon}$ (R2)
see (7.4) in [R10]
- the formulae expressing $\boldsymbol{\varepsilon}$ in terms of $\mathbf{q}$ (R3)
see (7.6) in [R10]

According to Clebsch theorem the matrix appearing in (R1) is the transpose of the matrix appearing in (R3). This means that the set of equations defining the strains determines directly the set of equations of equilibrium of nodes linking the internal forces with the effective loads. Thus, if the strains are expressed in terms of displacements/angles of rotation, there is no need to write special procedures to set the equilibrium equations, because their form is automatically constructed. Substitution of (R3) into (R2) and (R2) into (R1) leads to the governing equation: $\mathbf{Kq}=\mathbf{Q}$ with the symmetric and always positive semidefinite stiffness matrix $\mathbf{K}$. There is no need to construct element stiffness matrices and perform the relevant aggregation process, so widely described in the books on the finite element method. The typical courses on the finite element analysis of frames ignore the above system of equations thus neglecting the Clebsch theorem along with the overall elegance of the system (R1-R3). A detailed derivation of (R1-R3) within the Bernoulli-Euler theory can be found in [R10]; the extension to the Timoshenko theory has been done in [R5],

the only difference concerns the constitutive equations (the differences are slight) and the very definition of the internal forces. Despite taking into account the transverse shear deformations within the continuum 1D Timoshenko model the discretized unknowns involved in the system (R1-R3) are in fact the same. For instance, the transverse forces do not appear in the vector σ and the vector ε involves only the bending terms, the same as in the Bernoulli-Euler theory. This is due to a very specific mathematical structure of the Timoshenko system of equations.

The equations (R1-R3) are well known in the context of trusses (see e.g. Achtziger [R11]) and less known in the general context of frames in space. The set of equations (R1-R3) is fundamental being a discrete counterpart of the system of equations of statics of linearly elastic bodies.

Substitution of (R3) into (R2) leads to the mentioned transformation equations forming the operator called in the Thesis the Dirichlet-to-Neumann operators.

Let us summarize the main ideas of the Thesis. The optimization problems are considered for dynamical systems governed by the wave equation with the time horizon $T \rightarrow \infty$. The representative example is the model for Timoshenko bars composed of the linear system of hyperbolic equations. For the control problem with the tracking-type cost an abstract result on the so-called turnpike property is shown.

The abstract result on the exponential turnpike property gives the estimate between the solutions of optimality system for dynamic problem and the associated steady state problem. Practical implication of *turnpike property* is that the design of a network governed by dynamic state equations can be performed on the steady state equations. Therefore, the topological derivative method can be applied to design of networks. The proposed method is new, original and innovative. The singular perturbations of the network are considered for nucleation of small cycles. The domain decomposition method with the appropriate Steklov-Poincaré operators are applied for the purposes of modeling of the topological derivatives. Well-chosen numerical experiments confirm the theoretical results of the Dissertation.

The reviewer was informed that both the papers
Martin Gugat, Meizhi Qian, and Jan Sokołowski. Network design and control: Shape and topology optimization for the turnpike property for wave equation

Nikolai P. Osmolovskii, Meizhi Qian, and Jan Sokołowski. Net-work optimality conditions. Control and Cybernetics

have been accepted and will be published soon. A paper on Timoshenko bars is being now prepared.

4. Editorial remarks

The Thesis is not self-contained. Many non-typical conventions are adopted, some of them nowhere defined. A good example is the sign *hat* over a vector in space which rearranges the vector to the form of a skew symmetric matrix, see Sec.1.5, the last notation. It occurs that this notation is used only in Chapter 2, but the *hats* appear also later and then the *hat* over a font has nothing to do with the rearrangement

defined in Sec.1.5, it usually distinguishes the virtual fields from the unknown field, the latter is frequently written with the upper *hat*.

The notation *hat* in the meaning cleared up in Sec.1.5 has already been used by some authors, it is used for instance by Strohmeyer in his Doctor's Thesis [81]. Strohmeyer uses also the operation: *vec* inverse to the *hat* operation which transforms back a skew symmetric matrix to the vectorial form. The *vec* operation is also used in the Thesis reviewed (see Sec.2.1.1) but is not defined. The reader has no chance to guess that *vec* is the operation inverse to *hat*; this should have been cleared up in Sec. 1.5, but was not.

Notation is not unified. For instance, see page 79, the second derivative is denoted by two primes: $(\cdot)''$ (the fifth line from the upper) and two lines below the same second derivative is denoted by the upper index (2) in Eq.(5.20) and below.

The description of the theory of deformation of rods called *beams*, like in some non-engineering papers - let us remind that the notion: *beam* (*der Balken* in German, *belka* in Polish) suggests that the rod is only subjected to transverse loads in a plane, causing bending moment and transverse shear forces, while the axial forces and torsion should be zero - delivered in Chapter 2 is unclear; this theory has been expounded upon with sufficient details in the Thesis by Strohmeyer [81]. Let it be noted here that the theory of bars used in the Thesis reviewed has been previously developed in the sequence of papers which have been prepared with the aim of the computational implementations. The most famous expert in the theory of rods, Eric Reissner, contributed essentially to this topic but his numerous and valuable works (1950-1996) are being ignored by the younger generation of researchers. The aim of Reissner was to write equations as simple as possible, having in mind their future applicability for engineers. It is sad to note that these old masters' papers disappear and are replaced with formal texts with highly complicated equations, mostly written implicitly. Too much complicated equations will never be accepted by the engineering society, even if they are correct. Moreover, it is also sad to note that the valuable papers by Pietraszkiewicz, Makowski, Chróścielewski, Badur, Szwabowicz, see the books [R3] and Wiśniewski [R1] are also ignored; it is true that they mainly concern shells, not rods, but just the papers written in the late 1970's by Pietraszkiewicz (see [R4]) paved the way for the progress in the field of dealing with large rotations in shells and rods, continued later by Simo et al.

The bilinear form $a(u,v)$ which appeared at page 76 and involved in (5.6) and ((5.7) is nowhere defined. It is simply the integral of the product $u'v'$, but this information is missing.

Simplicity of the notation does not justify using the notation pf , thus using neither the dot nor the notation: $\langle \cdot, \cdot \rangle$ or $(\cdot, \cdot)$ for the scalar product of two vector functions, see page 22, the more that some authors dealing with optimum design, like A. Cherkaev and K. Lur'e, omit the sign of the tensorial product, hence some readers can think that pf represents $p \otimes f$. None the less the notation $\langle \cdot, \cdot \rangle$ appears in the Thesis in Sec. 3.1.2, 3.1.3, see e.g. Eq. (3.3).

The font Ω is not only used to denote domains but represents also the quadratic form, see Sec.3.1.2, which is non-standard.

The Ref.[62] is misleading. SIAM is here the Publisher of the book (not SIAM series of journals) printed in Philadelphia (this information should be added).

In Eq.(3.28) the differential operator should be: d/dx and not dd/ddx .

The same constant is written as: c_H or C_H , see Sec.3.2.

The Appendix A.4 is redundant, because the final result can be directly found by using the easy to use and well-known differentiation rule:

$$\frac{d}{dx} \int_{a(x)}^{b(x)} F(x,t) dt = \int_{a(x)}^{b(x)} \frac{\partial}{\partial x} F(x,t) dt + \frac{db}{dx} F(x,b(x)) - \frac{da}{dx} F(x,a(x)) \quad (R4)$$

In case of a problem in Sec A.4 we have: $a(x)=0$, $b(x)=x$, hence the second term equals $F(x,x)$ and the last term vanishes. In the case considered in Sec.A.4 we have $F(x,x)=0$ and the final result is reduced to the first term. The rule (R4) can be derived directly by using the fundamental properties of the integral operation and the differentiation rule for the compound functions.

5. Linguistic misprints

Overall the English language is correct. The minor mistakes are listed below

page	There is	There should be
iv	Lion's	Lions'
45	matrices are	matrix is
69, the caption of Fig.4.5	The static z ...	Displacements and angles of rotation ..
123	nauka, moscow, russian	Nauka, Moscow, Russian
123	Bendsoe	Bendsøe
127	russian	Russian
129	hilbert	Hilbert

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6. Conclusions

The Dissertation encompasses both the theoretical analyses as well as practical examples illustrating the methods and results, which is a great virtue of the work. To achieve this possibility the mathematical models are described within the theory of separable Hilbert spaces in which the bases are countable and the Galerkin method can be directly applied.

Due to one dimensionality of graphs the topological derivatives are constructed with using the exact solutions of the relevant sub-problems.

Worth mentioning is the unexpected formula for the topological derivative concerning frames composed of Timoshenko bars.

The main achievements of the Dissertation are:

- proof of the turnpike property
- complete optimality conditions for the nonlinear setting
- topological derivatives making it possible to introduce small cycles within a graph in order to improve the quality indicator
- numerical codes and illustrative numerical examples.

Concluding, the Thesis by Qian is interesting, modern, its multithreading is impressive. The Author has formulated and proved numerous valuable theorems concerning several topics: control theory, topology optimization and the theory of networks. The Author overmasters difficult skills of the theory of controllability of systems governed by nonlinear differential equations.

The Candidate is well prepared for an independent scientific research in the future.

On the basis of the above substantial arguments, I judge the work presented by Meizhi Qian worthy of a doctorate in Engineering Technology; the Dissertation satisfies all the requirements formulated in the Polish law: par.187 of the Act of July 20th, 2018: The Law on the High Education and Science (*Prawo o Szkolnictwie Wyższym i Nauce*; Dz. Ustaw 2018, p. 1668) with amendments introduced after July 20, 2018).

I am in favour of the defense of the Thesis before the Doctor's Jury of the Systems Research Institute, Polish Academy of Sciences in order to confer on Meizhi Qian the degree of Doctor in Engineering Technology.

Tomasz Lewiński

