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REVIEW

of the Ph. D. thesis of Ms. Meizhi Qian on
**Topological derivative method in control and design
of distributed parameter systems on metric graphs**

Supervisor: Prof. Jan Sokołowski
Co-supervisor Prof. Shengfeng Zhu

1 Introduction

1.1 Formal basis of the review

This review has been prepared on the basis of the resolution No. BA3-0004-6/23 of the Systems Research Institute Scientific Council as of 5th April 2024, regarding the appointment of reviewers of the doctoral thesis of Ms. Meizhi Qian in proceedings for the award of the Ph. D. degree in engineering sciences in the discipline of automation, electronics, electrical engineering and space technologies (letter of 24th April 2024 from the Deputy Director for Scientific Affairs).

1.2 Candidate for the Ph. D. degree

The author of the doctoral thesis is Ms. Meizhi Qian from the School of Mathematical Sciences, East China Normal University in Shanghai, carrying out the Ph. D. study at the Systems Research Institute of the Polish Academy of Sciences. Ms. Qian is the co-author of 6 papers, published or accepted, in international scientific journals and scientific conferences. The topics of the papers are in the areas of control and optimization theory, and numerical and computational methods.

1.3 General information on the Ph. D. thesis

The thesis is written in English and is entitled *Topological derivative method in control and design of distributed parameter systems on metric graphs*.

The supervisor is prof. Jan Sokołowski from the Systems Research Institute of the Polish Academy of Sciences, and the co-supervisor is prof. Shengfeng Zhu from the School of Mathematical Science, East China Normal University, Shanghai, China.

2 Discussion and evaluation of the results

2.1 Introduction and evaluation of the dissertation topic

The pipeline networks for transportation of gas, oil, water etc. and mechanical structures, like trusses and frames, made up of beams and rods, are common examples of networks whose design, control and optimization of operation are complex engineering tasks. Solutions to such

problems require the use of sophisticated mathematical and computational methods and for this reason it is an active area of intensive research.

The **goal** of the thesis is the development and application of a method based on the concept of the topological derivative to the optimization of the shape and topology of a network. An abstract network is modelled as a metric graph $G = \{V, E\}$, where V are the vertices and E are the edges. The graphs in the form of a tree and graphs with cycles are used. On the network, or more precisely, on the geometrical domain Ω , defining the network structure, some *dynamic state equations of the hyperbolic type*, both linear and non-linear, are defined. The main idea is to consider two-level optimization task. At the lower level, an optimal control problem on a time interval $[0, T]$, is solved and the dynamic optimality system is determined, say $(\hat{y}^T, \hat{p}^T, \hat{u}^T)$, which defines the optimal value of the cost functional $J_T(\hat{u}^T(\Omega))$. If for the optimality system the so-called *turnpike property* holds, then instead of the original dynamic optimality system one considers the steady state (equilibrium) optimality system, say $(\hat{z}, \hat{p}, \hat{v})$, which defines the optimal value of the related cost functional $I(\hat{v}(\Omega))$. Then, at the upper level one looks for the network structure Ω that optimizes the functional $J(\Omega) := I(\hat{v}(\Omega))$. For the shape and topology optimization of Ω the shape and topological derivatives can be used.

In this context, I consider the topic of research undertaken by Ms. Meizhi Qian in her doctoral dissertation to be significantly important from the point of view of the development and applications of the theory of shape and topological derivatives to control and optimization problems defined on networks.

2.2 Discussion of the contents of the thesis

The dissertation consists of 187 pages, comprising the title page, an abstract, list of publications, a table of contents, six chapters, a bibliography with 83 entries, an acknowledgement and an appendix with Matlab codes used in numerical computations.

Chapter 1 *Introduction* contains a short literature review, motivation and presentation of the main idea of the problem to be investigated. Moreover, it sketches meanings of optimal control problems governed by partial differential equations, modelling, control and design of networks, the notation used throughout the work and finally, outlines the main results.

Chapter 2 *Modeling of Beams* presents a description of mathematical models under consideration, covering the geometrically exact beams model and the Timoshenko beams model, governed by semilinear hyperbolic systems. In the thesis these **dynamic models**, also called **evolution models**, are restricted to one-dimension in space which leads to ordinary differential equations at equilibrium states (steady states). These equations are called the **static models**, or **steady state models**, and, on a single edge of the graph, they are parametrized by one spatial variable.

In **Chapter 3** *Necessary and Sufficient Optimality Conditions for the Nonlinear Model*, the quite general optimization problems for the nonlinear steady state models are considered. The justification for the replacement of the evolution state equations by the static steady state equations is the phenomenon called the *turnpike property*. The nonlinear models considered in this chapter, i.e. the quasilinear hyperbolic systems and the semilinear hyperbolic ones, cover, among others, the gas and hydrogen distribution networks. The optimality conditions are obtained by means of the local Pontryagin principle. Both, the control and shape problems are dealt with within the same mathematical framework of necessary and sufficient optimality conditions. Two numerical examples, of a simple network with one edge, illustrate the practical applicability of the obtained theoretical results.

Chapter 4 *Static Optimization for Timoshenko Beams* studies the optimal control problem on the network with static mathematical models of Timoshenko beams, defined on the edges of the network, and the tracking type quadratic cost functional without constraints. The necessary

and sufficient optimality conditions for the static optimization problems are obtained by using the Lagrange formalism. The shape gradient of the optimal cost and topological derivative are determined. New results on the topological derivative of the shape functionals for Timoshenko beams are derived by using the domain decomposition techniques combined with the asymptotic analysis of the Steklov-Poincare operator. The small parameter in the analysis corresponds to the small cycle introduced at some internal node of the network. Thus, the topology optimization of the network is performed by nucleation of the small cycles. Finally, some numerical examples for both models are computed to demonstrate applicability of the results in a more practical environment.

In **Chapter 5** *Turnpike Property for Dynamic Optimization and Applications* two, corresponding to each other, optimal control problems for the scalar wave equation are considered. In both problems the cost functionals are quadratic and defined by

$$J_T(u) = \frac{1}{2} \int_0^T \|y - y^d\|_{L^2(\Omega)}^2 + \gamma \|\partial_t(y - y^d)\|_{L^2(\Omega)}^2 + \frac{1}{2} \int_0^T \|u - u^d\|_{L^2(\Gamma)}^2 dt$$

for the evolution wave equation, where $\gamma > 0$ and $T > 0$, and by

$$I(v) = \frac{1}{2} \|z - z^d\|_{L^2(\Omega)}^2 + \frac{1}{2} \|v - v^d\|_{L^2(\Gamma)}^2$$

for the static wave equation, i.e. the differential equation characterizing the equilibrium, or steady state, of the evolution equation. The solutions to both differential equations are understood in the weak sense. The differences

$$\omega^T = \hat{y}^T - \hat{z}^\sigma, \quad \mu^T = \hat{p}^T - \hat{p}^\sigma, \quad \nu^T = \hat{u}^T - \hat{v}^\sigma,$$

between the dynamic optimality system variables $(\hat{y}^T, \hat{p}^T, \hat{u}^T)$ and the static optimality system variables $(\hat{z}^\sigma, \hat{p}^\sigma, \hat{v}^\sigma)$ are introduced and their properties for the networks in the form of trees are analyzed. The original result of this chapter shows that the differences satisfy the so-called *turnpike property* in the form

$$\|\omega^T(t)\|_{L^2(\Omega)}^2 + \|\mu^T(t)\|_{L^2(\Omega)}^2 + \|\nu^T(t)\|_{L^2(\Omega)}^2 \leq D(e^{-\sqrt{\gamma}t} + e^{-\sqrt{\gamma}(T-t)}), \quad t \in [0, T],$$

where D is independent of T . It also implies that $\lim_{T \rightarrow \infty} \frac{1}{T} J_T(\hat{y}^T) = I(\hat{v})$. The turnpike property justifies the approximation of the dynamic optimization problem by the static optimization problem and for this reason the shape and topology optimization is easier to handle. In the final part of this chapter two numerical examples are presented for the topological derivative of the optimal cost evaluation for the network singular perturbations by nucleation of small cycles. It is shown how the optimal size of the cycle is computed.

The thesis final **Chapter 6** entitled *Conclusions and Further Research* briefly sums up the main achievements and proposes further problems of interest to pursue in the future research. These include the research on turnpike properties for new problems and topological derivatives for networks with new equations.

The dissertation includes **Bibliography** comprising 83 entries, all related to the dissertation topics or containing a description of the mathematical tools used. This inventory is a reasonably complete compendium of knowledge in the dissertation's subject area.

Several numerical examples have been worked out in Chapters 3-5 in order to illustrate practical applicability of the theory. All computations have been performed with the MATLAB software and detailed codes are listed in the **Appendix**, counting 46 pages.

The notations and terms of language used in the thesis are appropriate and the editing is satisfactory. The author uses precise language, employing appropriate vocabulary, and the considerations and reasoning are carried out in a logical manner. Before formulating the problem, the author provides a brief introduction and then clearly poses and solves the problem under consideration. The dissertation can be considered an example of a high level doctoral thesis.

2.3 Evaluation of the results

The results realizing the goal of the thesis are contained in Chapters 3-5.

The main result of Chapter 3 is a **non-trivial application** of the local Pontryagin's maximum principle to the formulation of the necessary optimality conditions for the nonlinear static beams. As a valuable support of the theory two simple numerical examples are fully worked out with computations performed within the Matlab environment.

In turn, in Chapter 4, a **comprehensive new description** of how to handle static optimization for a network of Timoshenko beams is presented. For a network there are two possible variations of the shape Ω . One is to change of the lengths of edges and corresponds to the boundary variations in the classical shape optimization. The other corresponds to the topology variations and corresponds to nucleation of a small cycle within the network with the small size $\epsilon \rightarrow 0$. It can be summarized as follows: There exists a unique solution $(\hat{z}, \hat{p}, \hat{v})$ to the optimality system, defined by equations (4.4)-(4.6). The optimal value of the cost $J(\hat{v}) := J(\hat{v}(\Omega))$ is defined as a shape functional over the set of admissible shapes S_{ad} and the optimum design of the network is given as $\inf_{\Omega \in S_{ad}} J(\hat{v}(\Omega))$. By performing the shape calculus the shape gradient of the optimal control cost $\Omega \mapsto J(\hat{v}(\Omega))$ and the topological derivative at $\epsilon := 0^+$ for the mapping $\epsilon \mapsto J(\hat{v}(\Omega_\epsilon))$, are determined. It is worth to emphasize that the results in this chapter will appear in the journal *Control and Cybernetics*.

The most valuable and original result of Chapter 5 is the **extensive proof of the turnpike property** for the optimality system for the wave equation on the network. In fact this result allows for approximation of the shape optimization problem for dynamic equations by the corresponding problem for the static equation and in this way it makes the solution of the network shape optimization feasible. The results of this chapter have been submitted for publication in the *Journal of Geometric Analysis*.

In conclusion, the results of Chapters 3-5 as a whole establish themselves a new method of the network shape optimization for distributed parameter control systems with networks defined on metric graphs, and I believe that the **thesis goal**, formulated by the Author in **Chapter 1** and quoted above in Subsection 2.1, **has been fully realised**.

2.4 Remarks and comments

1. In general, the dissertation is well written and free of mathematical errors. At some places there are few misprints, and by mistake on pages 2 and 75 instead of the term *turnpike property* the term *turnpike poverty* is used.
2. In the final chapter the Author outlines interesting important directions of possible further research.
3. Here I think it would be interesting to extend the developed method to distributed parameter systems modelled as the port-Hamiltonian ones. This class of models well reflects the physical properties of systems and is well suited for linear and non-linear hyperbolic equations.
4. Another interesting question is whether the turnpike property would stay invariant under the Cayley-Tustin transformation which transforms time-continuous infinite-dimensional systems to discrete-time infinite-dimensional system and preserves several system theoretic properties including stability, controllability etc.

3 Final conclusion

The comments made in the Subsection 2.4 are of a debatable nature and do not affect my overall **positive** opinion of the dissertation. The reviewed work has all the necessary characteristics of a doctoral dissertation in the field of engineering sciences in the discipline of automation, electronics, electrical engineering and space technologies:

- **It demonstrates the author's extensive theoretical knowledge** of modelling, control and optimization theory and computational methods for distributed parameter systems, including theory of shape and topological derivatives, and it also demonstrates author's ability to put this knowledge to practical use in independent research.
- It includes the **formulation and solution of an original scientific problem** that significantly complements and extends the existing knowledge (in this case, it is the development and application of a method based on the concept of the topological derivative to the optimization of the shape and topology of networks defined on metric graphs, including the proof of the turnpike property for the optimal control of the wave equation).

In final conclusion, I clearly state that the dissertation falls within the current research area of control and optimization theory and computational methods for distributed parameter systems, which are important branches of the scientific discipline **automation, electronics, electrical engineering and space technologies**, and it **fulfils** all the requirements set out in Art. 187(1-3) of the Act of 20 July 2018 *Law on Higher Education and Science*, Journal of Laws of 2023, item 742 as amended (in Polish: Art. 187 ust. 1-3 ustawy z dnia 20 lipca 2018 r. *Prawo o szkolnictwie wyższym i nauce*, Dz. U. z 2023 r. poz. 742 ze zm.). On this basis, I **request to admit** the dissertation for public defence.

