

Outer approximations scheme with quadratic cuts for weakly convex optimization problems

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Abstract of the Thesis

Outer approximation methods have been used to solve linear programming problems in the 60s and are still employed to solve variational inequalities problems, general convex problems, mixed integer linear and nonlinear programming problems.

In this work, we propose a novel outer approximation scheme for solving weakly convex optimization problems. The main idea is to use quadratic cuts instead of linear cuts and solving at each iteration a nonconvex QCQP problem. We investigate Lagrangian duality and convex relaxation for nonconvex QCQP. We show that the outer approximation problem belongs to the class of S-QCQP, where the quadratic terms of the objective function and the constraints are formed by a (possibly negative) scalar multiplied by the normed squared. We establish necessary and sufficient condition for global optimality of S-QCQP, under certain assumptions. The main result of this work is that every convergent subsequence produced by the outer approximation scheme proposed converges to the global minimum of a general weakly convex optimization problem.

The algorithm is applied to the circular packing problem, which can be reformulated as a weakly convex constrained problem.

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1 Scientific CV

1. Master degree gained, at La Sapienza, University of Rome, on 24/07/2020 the master's degree in INGEGNERIA GESTIONALE [LM - Ordin. 2016] (classe LM-31), (Management Engineering).
2. Joined TraDE-OPT project (<https://trade-opt-itn.eu/>) as PhD student at IBS PAN in October 2020.
3. Secondment in Centre de Vision Numérique, University Paris-Saclay from October 2022 to December 2022.

Giovanni Bruccola gave a talk based on the paper "On global solvability of a class of possibly nonconvex QCQP problems in Hilbert spaces" at

- Modern Techniques of Very Large Scale Optimization, Edinburgh, May 2022,
- TraDE-OPT workshop on Algorithmic and Continuous Optimization, Louvain-La-Neuve, July 2022,
- GPCO, Apolda, September 2022.

Giovanni Bruccola gave a talk based on "Calculus rules for proximal ε -subdifferentials and inexact proximity operators for weakly convex functions" at

- European Control Conference (ECC), Bucharest, June 2023.

Other conferences attended

- Wombat, on line, December 2021,
- ITN winter school, on line, February 2021,
- ITN summer school, on line, July 2021,
- Wombat, on line, December 2022.
- TraDE-OPT (<https://trade-opt-itn.eu/>) Workshop as PhD student, at Graz, in March 2023.

2 List of publications

- Bednarczuk, E. M., Bruccola, G. (2023). On global solvability of a class of possibly nonconvex QCQP problems in Hilbert spaces. Optimization, 1-20.
- Bednarczuk, E., Bruccola, G., Scrivanti, G., Tran, T. H. (2023, June). Calculus rules for proximal ε -subdifferentials and inexact proximity operators for weakly convex functions. In 2023 European Control Conference (ECC) (pp. 1-8). IEEE.
- Bednarczuk, E., Bruccola, G., Scrivanti, G., Tran, T. H. (2023). Convergence analysis of an inexact Forward-Backward algorithm for problems involving weakly convex functions. arXiv preprint arXiv:2303.14021. Submitted to journal.
- Bednarczuk, E., Bruccola, G., Pesquet, J. C., Krzysztof E. Rutkowski (2024). Outer Approximation Scheme for Weakly Convex Constrained Optimization Problems. In preparation.