Warszawa, 20.09.2024

dr hab. Mariusz Białecki, prof. IGF PAN
Department of Theoretical Geophysics
Institute of Geophysics, Polish Academy of Sciences

Review of the PhD Thesis

"Affine continuous cellular automata and their role in solving density classification problems"

by Marcin Dembowski

The doctoral dissertation of Marcin Dembowski was prepared in System Research Institute, Polish Academy of Sciences, under supervision prof. dr. Bernard De Baets from Ghent University, and co-supervision of dr. Barbara Wolnik from University of Gdańsk.

The dissertation consists of the results published in the following publications:

1. B. Wolnik, M. Dembowski, W. Bołt, J. M. Baetens, and B. De Baets. The Density Classification Problem in the Context of Continuous Cellular Automata. In: Cellular Automata: 12th International Conference on Cellular Automata for Research and Industry, ACRI 2016, Fez, Morocco, September 5-8, 2016. Proceedings. Ed. By S. El Yacoubi, J.Wąs, and S. Bandini. Springer, 2016, pp. 79–87.
2. B.Wolnik, M. Dembowski,W. Bołt, J. M. Baetens, and B. De Baets. Density-conserving affine continuous cellular automata solving the relaxed density classification problem. In: Journal of Physics A: Mathematical and Theoretical 50.34 (2017), p. 345103.
3. M. Dembowski, B. Wolnik, W. T. Bołt, J. Baetens, and B. De Baets. Affine continuous cellular automata solving the fixed-length density classification problem. In: Natural Computing 17.3 (2018), pp. 467–477.

4. M. Dembowski, B. Wolnik, W. T. Bołt, J. Baetens, and B. De Baets. Two-dimensional affine continuous cellular automata solving the relaxed density classification problem. In: Journal of Cellular Automata 14.3-4 (2019), pp. 191–212.
5. B. Wolnik, M. Dembowski, A. Augustynowicz, and B. De Baets. A complete description of the dynamics of legal outer-totalistic affine continuous cellular automata. In: Nonlinear Dynamics 110.1 (2022), pp. 589–610.

The thesis is written in English on 134 pages (plus 14 pages of Front Matter and 8 pages of Back Matter). The main part, consist of 10 Chapters and is a unified presentation of the contents of 5 articles co-authored by Marcin Dembowski, enriched with general introduction and explanation of the topic. In particular, Chapter 5 contains the results of article 5, Chapter 6 – results of the conference paper 1 and article 2, Chapter 7 – article 3, Chapter 8 – article 4, and Chapter 9 contains non published material. The last chapter There is 100 references cited through the text and short list of notation. The thesis contains also QR-codes sending a reader to websites with examples and nice visualisations.

As claimed in Preface, Marcin Dembowski authored all visualizations, simulations, experiments in this thesis, published articles and presentations and analyzed the results collected from experiments that were a base work for further analytical proofs. Proofs of the theorems were achieved in cooperation with Barbara Wolnik (all papers) and Antoni Augustynowicz (the last paper). The remaining authors supported the research by discussions, validations and corrections.

The results were presented at the following international conferences: Automata 2015 - presented by Witold Bołt, ACRI 2016, Summer Solstice 2017, 2018, Automata 2017, 2018, 2019 - presented by Marcin Dembowski, and Summer Solstice 2022 - presented by Antoni Augustynowicz.

The doctoral thesis deals with the widely known Density Classification Problem, and more specifically the use of a subclass of Continuous Cellular Automata (CCA), namely Affine Continuous Cellular Automata (ACCA) to solve this problem.

Cellular Automata (CA) are dynamical systems for which all independent variables (space,

time) and dependent variables (cell states) are discrete. Continuous Cellular Automata can be seen as a generalization of CA – dependent variables (cell states) can take values from any subset of real numbers. Using other terminology (used for example in the context of integrable systems), CCAs refer to discrete functions, while CAs refer to ultradiscrete functions. ACCAs form a subclass of CCAs for which the local rule is affine, i.e. of the form ax_i+b in each variable x_i, where a and b are some real coefficients. There are various subclasses of ACCAs, like Outer-totalistic ACCAs, Density-conserving ACCAs, Rotation-symmetric ACCAs, characterized in detail in Chapter 3 of the thesis.

The main subject of the thesis is introduced in Chapter 4. Since it is known, that for any dimension d there is no d-dimensional binary CA that solves the Density Classification Problem, thus potential candidates were sought in the broader CAs classes. Consideration of ACCAs for solving the problem is natural and justified step. It is shown that the density classification problem in its classical formulation has no solution in the class of ACCAs. (I have a critical remark here - in the reasoning of the proof, a property is used that has not been proven, but a reference has been indicated. Due to better readability, it would be worth presenting its justification in the thesis, even in the Appendix.) Thus it is legitimate to modify/relax the original problem (see Definition 4.3.1) and then to investigate it in this new context for respective solution. This constitute most of the remaining text of the thesis.

Chapter 5, based on the chronologically most recent publication, describes in complete terms the dynamics of One-Dimensional Legal Outer-Totalistic ACCAs. This significant result was achieved through a combination of extensive computer simulations and the appropriate application of various analytical methods. This chapter shows the benefit of using well-planned and executed (numerous) numerical simulations to obtain an important exact mathematical result. The methods presented here clearly indicate high scientific qualifications.

The solution to the relaxed one-dimensional DCP is elegant. Analysis of the convergence time (for a given size) based on numerical experiments is a valuable addition giving insight in the computational abilities of CCAs. Based on these results, the Relaxed Two-Dimensional DCP

was investigated, which is definitely challenging, especially from the computational side. In this case, extensive simulations were made for the 5 x 5 and 23 x 23 sizes in order to find the rules with the best convergence time.

The remaining part of the thesis is devoted to the investigation of Fixed-Length DCP. For a one-dimensional case necessary conditions for a rule to solve the DCP (for a given number of cells N) were derived. The formulation of sufficient conditions in this case remains an open problem. It is worth noting that the methods used to establish these conditions have made it possible, as it were, to draw interesting general conclusions, namely Corollary 7.2.3 and Corollary 7.2.4. This speaks very positively about the way research is conducted. Appropriate exemplary simulations were carried out for size N=23, which showed that the set of solutions have a non-trivial structure and contain rules greatly differing in convergence time.

The Fixed-Length DCP in two dimensions is reformulated as an optimization problem and is investigated in numerical experiments aided by evolutionary search methods. Appropriate rules of evolution are described and applied to experiments for various (small) sizes of the grid. Those experiments shows that finding ACCA rules that solve the Two-Dimensional DCP is possible, however it require smart optimization for reducing the number of parameters in the search. The experience obtained allowed the formulation of several hypotheses to be tested in the future.

The doctoral thesis concludes with a few interesting guidelines for further research.

The whole thesis is written in a careful and legible manner. The maturity of the way of presentation deserves praise, both in the overall (organization of content) and local (presentation of reasoning) approach. The drawings are very nice and legible. The entire graphic layout of the thesis is very elegant. If the author would like to make the presented text more widely available as an introduction to the subject matter they are developing, I would suggest supplementing it with an index (useful for novices) and presenting all arguments in the proof of the Theorem 4.2.1, instead of indicating the references, as I mentioned earlier. Simulations were carried out for size N=23, which showed that the set of solutions have a non-trivial structure and contain rules greatly differing in convergence time.

Assessment of the candidate's general theoretical knowledge in the discipline and the ability to independently carry out scientific research

The dissertation demonstrates that the candidate has the wide and deep knowledge of cellular automata both in theoretical and computational aspects.

The doctoral dissertation presents the results of systematic, well-planned and extensive research on the role of Affine Continuous Cellular Automata in solving Density Classification Problems. It contains important results in the form of mathematical theorems, and presents the results of skilfully tailored numerical experiments. Some of the theoretical results were created on the basis of extensive simulations.

The candidate proved, that he can successfully share obtained results both by writing and publishing scientific articles and doctoral thesis as well as by giving oral presentations. He also is experienced in international collaboration.

The justification that the solution of the problem in the doctoral dissertation is original

To my knowledge, the solution to the problem in the presented doctoral dissertation is original. In addition, the results were originally published in 5 scientific articles in monographs and scientific journals by the group of authors as indicated earlier, in all of which the candidate is the first or the second author.

Opinion regarding the candidate's admission to the public defence of the doctoral dissertation

I find the dissertation as satisfying the requirements for PhD thesis and on this basis the candidate deserves to be admitted to the public defence of a doctoral dissertation.

Distinction

I consider the subject of the dissertation to be interesting and important for the field of computational capabilities and dynamic characteristics of Continuous Cellular Automata. In my opinion, the use of diversified approaches and techniques to investigate the topic presented by the candidate far exceeds standard requirements. The obtained results are scientifically sound, which is why I propose this dissertation for distinction.

