

Abstract Convexity: Duality, Primal-Dual and Proximal based Algorithms

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Abstract: In this thesis, we investigate a class of nonconvex functions with special structure, which we call abstract convex functions. They are defined on the basis of generalized Fenchel conjugation and bi-conjugation with arbitrary coupling functions (or elementary functions). When the coupling functions are affine, we recover the original convex functions. Abstract convex functions are accompanied by abstract conjugate and abstract subdifferentials which are global. Consequently, duality theory for minimization of abstract convex functions is not the classical one, in general, it is not related directly to the dual space. This fact is at the core of our studies of conjugate and Lagrange dualities for minimization problems involving abstract convex functions, which is the first task of the thesis. While the theory of abstract convexity has been developing since ninetieth, there is a lack of algorithms and real-life applications of abstract convex functions. One reason is the non-linearity of elementary functions which makes it difficult to design practically workable algorithms and investigate their convergence. The second task of this thesis is to generalize and adapt some of the algorithmic ideas functioning efficiently in the context of convex optimization to a specific class of abstract convex functions which we call $\Phi_{lsc}^{\mathbb{R}}$ -convex. Finally, we connect the first and the second part of our results to propose $\Phi_{lsc}^{\mathbb{R}}$ -primal-dual algorithm which exploits the idea of Chambolle-Pock algorithm.

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Papers submitted:

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- Bednarczuk, E., Brucola, G., Scrivanti, G., and Tran, T. H. (2023b). Convergence analysis of an inexact forward-backward algorithm for problems involving weakly convex functions. *arXiv preprint arXiv:2303.14021*
- Bednarczuk, E., Brucola, G., Scrivanti, G., and Tran, T. H. (2023a). Calculus rules for proximal ε -subdifferentials and inexact proximity operators for weakly convex functions. In *2023 European Control Conference (ECC)*, pages 1–8. IEEE
- Bednarczuk, E. and Tran, T. H. (2023). Duality for composite optimization problem within the framework of abstract convexity. *Optimization*, 72(1):37–80

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