

prof. dr hab. Stanisław Migórski

Jagiellonian University in Krakow, Faculty of Mathematics and Computer Science
ul. Łojasiewicza 6, 30-348 Krakow, tel. 12 664 6666, fax. 12 664 66 73

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Evaluation of the doctoral dissertation of Ms Yixin Tan, MSc
entitled
“Control and Optimum Design of Distributed Parameter Systems.
Gradient Flow and Applications to Elasticity Systems”
for
Scientific Council of Systems Research Institute
of Polish Academy of Sciences

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The doctoral dissertation of Ms Yixin Tan is a comprehensive study on the analysis and numerical solution of optimization problems in the theory of elasticity, encompassing boundary control, shape and topology optimization, as well as problems involving contact and nonlinear constitutive relations. The goal of the thesis is to formulate these problems within a consistent variational framework, combining the modelling of partial differential equations, sensitivity analysis, and algorithmic implementation.

Overview of the dissertation. I will begin with a discussion on the core part of the dissertation. The thesis consists of five chapters and has a references list of 67 items. Chapter 1, after presenting a motivation, describes challenges which involve boundary control, shape and topology evolution, nonlinear elasticity, contact mechanics, and numerical studies.

Chapter 2 is devoted to a systematic study of the boundary control problem of linear elasticity with a frictional contact boundary condition. First, the author recalls the optimal control problem for the contact model with a quadratic cost functional. Based on the Karush-Kuhn-Tucker conditions, the first-order optimality system is presented. It involves state and adjoint equations in their weak form and an optimality condition. Next, the shape sensitivity analysis is discussed. Its goal is to study how the objective functional changes under the perturbation of the domain. To generate a smooth deformation of the domain, a family of small Lipschitz perturbations of the identity map is employed. Subsequently, we are led to the examination of shape gradients, topological derivatives, and the study of topological sensitivity. These results are also extended to the analysis of a unilateral frictional contact problem. Further, an important concept of the turnpike property is mentioned that describes how optimal solutions to a dynamic

problem tend to stay close to a certain “steady state” for most of the time horizon. It should be noted that most of the results are presented with no proof but in a clear way. Numerical examples based on the shape gradient conclude the section.

Chapter 3 examines the class of contact problems in mechanics with the Signorini boundary condition on a potential contact part of the boundary. Here, the author applied a phase-field method to regularise a nonsmooth contact problem being a variational inequality with unilateral constraint. The idea is to remove the need to represent explicitly the contact boundary by introducing an auxiliary continuous function, the so-called phase field. In this way the interface between materials is diffused and approximated by the level set of this function. This regularization leads to a continuous contact energy function which facilitates the sensitivity studies and suits well to shape and topology optimization. For the reader’s convenience, it would be helpful to briefly recall the concept of Mosco convergence of sets. An optimization approach is discussed, and it combines the phase-field method with a topological derivative. The section concludes with a consideration of frictional contact problems and some numerical examples.

Chapter 4 differs from the previous parts. It deals with a discussion of gradient flows arising in shape optimization for PDEs. The material follows the results due to Plotnikov and Sokolowski. Here, gradient flow refers to a type of evolution PDE. In the study of gradient flows, we are looking at how a point evolves over time to minimize a functional. This is formalized as a differential equation that follows the direction of steepest descent of a functional. Two models of shape optimization are examined: an inverse problem for the Laplace operator in the plane, and an interface identification problem in linear elasticity. The numerical results of the chapter are new.

Chapters 3 and 4 are, in my opinion, the most important parts of the dissertation. They deal with different approaches and are significant from the point of view of applications. Applications may include compliance minimization, inverse interface identification problems, and shape reconstruction. The author’s contribution is further strengthened by the inclusion of two examples in Chapter 3 and four in Chapter 4, all featuring explicitly computed solutions that effectively illustrate the theoretical results. Numerical experiments confirm the decrease of the cost functional, and compared to the classical level-set approach, not only better stability but also convergence was achieved.

In Chapter 5, the author makes an attempt to generalize the former shape sensitivity analysis to a stationary nonlinear elastic system. An application of the discontinuous Galerkin FEM and two numerical examples for nonlinear beams are presented.

The thesis also presents future research directions. An important component of the dissertation is represented by a numerical implementation of the problems under consideration. The thesis is supplemented by a computer code dealing with computational issues.

Evaluation of the dissertation. The dissertation is of a high scholarly standard. It refers to current publications in the field of shape optimization, control and optimum design of distributed parameter systems. The necessary theoretical results from the recent contributions of Plotnikov and Sokolowski are recalled for the purpose of computational

methods. The numerical methods in the thesis are an original, new and important contribution of the author.

The author of the dissertation has demonstrated a very good understanding of the subject of the undertaken research, as well as strong proficiency in the use of tools from mathematical analysis. The problems considered are analytically and computationally challenging due to their complexity and have been approached in a thorough manner. The dissertation has a clear and logical structure and has been prepared with great care. Selected results obtained may form the initial phase of a rich scientific research programme.

The dissertation is written clearly, and the shortcomings in the text are very minor. Some minor typos mentioned below do not have any impact on my positive opinion on the work.

page 26: In (2.37) the function $\mathbf{f}_1$ is not defined.

page 43: “equationm”.

page 55: “for ethe lastic”, “system fora”, “cousidered”, “optimam”, “selested”, “counplience”.

page 56: “perturbetions”, “usedd”. The third sentence on this page is not complete.

In certain sections, particularly in Chapter 4, it would be beneficial for a reader to include additional references to related work and to more clearly highlight the novel contributions of the thesis in comparison with existing literature. Despite the minor comments included in my opinion, I find that the entire doctoral dissertation is prepared and written at a good level.

Final conclusion. In conclusion, I assess the doctoral dissertation highly. The contribution of the work is both analytical and computational. Analytically, the gradient-flow-based approach provides a framework for shape optimization problems with PDE constraints, grounded in shape calculus and variational evolution theory, with convergence guarantees based on energy dissipation arguments. Computationally, the integration of phase-field methods, topological derivatives, and discontinuous Galerkin discretization forms a mathematically consistent and numerically stable toolkit for interface problems, inverse elasticity problems, and structural design.

The author has demonstrated a deep understanding of the research topic, high mathematical competence, and the ability to apply advanced analytical and computational tools to complex problems. In summary, the dissertation meets both the formal and substantive requirements for doctoral theses as defined by the relevant academic regulations.

In conclusion, the dissertation fulfils the necessary criteria and the required academic standards and makes a valuable contribution to the field. Therefore, I give this thesis a positive recommendation and support the acceptance of the dissertation for further stages of the doctoral procedure.

**Stanisław
Migórski**

prof. dr hab. Stanisław Migórski

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