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Review of the doctoral dissertation of **Yixin Tan**, MSc,
employed at East China Normal University, entitled

*Control and Optimum Design of Distributed Parameter
Systems.
Gradient Flow and Applications to Elasticity Systems*

prepared on order from Systems Research Institute of the Polish Academy
of Sciences.

General description. The dissertation starts with the Introduction, which contains the enumeration of the considered problems and the motivation, which is stated as exploring various numerical approaches to shape optimization for elastic structures. The author states also the expected results.

Chapter 2 contains the description of weak and variational formulations of elastic systems and statement of relevant control problems. Furthermore the tools of shape analysis (shape derivative, topological derivative) are introduced. Some examples 2D are solved using these techniques various formulations like for example phase field and variational inequalities. The dynamic control problems are analyzed in theoretical manner.

Chapter 3 deals with phase field modeling of contact problems. What I consider as interesting, is applying the shape calculus to both system and adjoint variables simultaneously. One of the main results here is the rigorous treatment of the elastic contact with approximated friction term.

Chapter 4 introduces gradient flow theory for shape optimization of static cases. It contains some theoretical background examples of problems which may be treated in this way. By introducing artificial time the optimization process may be treated as dynamical system and its convergence may be analyzed. Here the main achievement consists in obtaining the relevant formulas, which may be used in numerical examples.

Chapter 5 concerns analysis of nonlinear materials using the same methods as in previous Chapters. The example also illustrates the inherent non-uniqueness of optima in shape design.

Main achievements. In my opinion the main achievements of the dissertations are;

1. Collecting various shape optimization problems and treating them using the same set of techniques, namely shape calculus, topological derivative, and gradient flow approach. It required very high level of sophistication in mathematical and functional analysis.
2. I personally consider as achievement simultaneous treatment of optimality system (state and adjoint) with tools used in shape optimization.
3. In particular cases there were obtained rigorous results concerning forms of optimality systems, which required very high skills in analysis.
4. Solution of several nontrivial examples. The examples used for illustration of the gradient flow method are especially worth mentioning, because they are the first of this kind of which I am aware.

In general the thesis contains a great deal of material showing maturity of the candidate in the field of mathematical analysis and numerical methods. Some of the derived formulas are new. The main gist of the work lies in theoretical derivation of rigorous formulations, but the numerical part is also nontrivial.

Remarks. The thesis is generally well written and carefully checked with respect to style. There are some repetitions, like introducing elasticity system with the same notation several time, but it probably improves clarity of presentation. This is important, because the variety of problems and applied techniques make for very dense reading. My remarks are of slightly more general nature.

1. The concluding remarks mention the possibility of using static solutions and turnpike property for speeding up computations. Is there any possibility to prove turnpike property for problems under consideration ?
2. In examples for Chapter 3 the solutions for cases with and without friction look very similar. Is it caused by the fact, that introducing strong tangent stresses caused numerical difficulties ?
3. In phase field approach one uses a two-well energy potential depending on phase variable ϕ , the value of which parameterizes the mixture of

weak and strong materials. The dynamics of phase variable depends on artificial time and stresses of displacements and adjoined variable, which leads to phase separation. How it differs from SIMP methods, where we have post-processing, which also may be viewed as some kind of phase separation.

4. There are some misprints in Theorem 4.7.1 (e.g. $\lambda \geq r$?), as well as wrong back references. On page 179 there is reference to Section X.X.
5. Examples for gradient flow approach are very smooth: no boundary conditions changes, no corners, infinitely smooth boundaries. Can you foresee any progress in this respect ? Is it necessary ?

Conclusions. Taking into account all comments and observations listed above, I express the opinion that the Dissertation of **Yixin Tan** satisfies all the requirements of appropriate enactments for obtaining Ph.D. degree (art. 187 Ustawy z dnia 20 lipca 2018 r. - Prawo o szkolnictwie wyższym i nauce (Dz.U. 2018 poz. 1668 z óżn. zm.)). I recommend acceptance of the Dissertation for public defense.

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