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1. Review of the doctoral dissertation of M.Sc. Yixin Tan

Control and Optimum Design of Distributed Parameter Systems. Gradient Flow and Applications to Elasticity Systems

1. General remarks

This review was prepared at the request of Dr. Eng. habil. Jan Owsinski, Deputy Director for Research of the Systems Research Institute of the Polish Academy of Sciences (letter dated July 9, 2025).

The doctoral dissertation by M.Sc. Yixin Tan concerns optimal control, shape optimization, and topology optimization in elasticity problems. The candidate focuses on developing a unified approach that combines mathematical analysis with computational methods. The dissertation covers linear and nonlinear elasticity, contact problems, and optimal structural design.

Undertaking this research topic is well-justified given the continued importance of shape and topology optimization in the context of a coherent mathematical description within optimal control.

The work was carried out at the Systems Research Institute of the Polish Academy of Sciences under the supervision of Prof. Jan Sokołowski. The co-supervisor is Dr. Shengfeng Zhu, School of Mathematical Sciences, East China Normal University, China.

Given the aims and scope of the dissertation, it can be classified within the discipline of Automation, Electronics, Electrical Engineering, and Space Technologies.

2. Scope of the dissertation

The dissertation is written in English and consists of 206 pages. It includes abstracts in English and Polish, five chapters, a conclusion, listings of two computer programs, a bibliography, and acknowledgements.

Chapter 1 presents the motivation for the research and the notation adopted in the dissertation.

Chapter 2 deals with optimal boundary control in linear elasticity, where surface forces are treated as controls. By formulating the problem via the state equation, the adjoint equation, and optimality conditions, and by exploiting the turnpike property for the control problem, the dynamic problem is reduced to a static one, which significantly lowers the computational cost.

Chapter 3 addresses contact problems. Two strategies are proposed: phase-field regularization for smooth interface modeling and topological derivatives for singular perturbations of the geometric domain. These two methods are combined into a flexible approach to optimization in the presence of contact. The chapter also examines gradient-based formulations for static contact, in which using the elastic energy as a shape functional ensures consistency despite the nonsmooth contact model.

In Chapter 4, the candidate studies gradient flow for shape and topology optimization. This approach treats the optimization process as a dynamical system driven by shape derivatives, where velocity fields determine the evolution of the boundary. It introduces curvature regularization and an adaptive step-rejection mechanism, which guarantee a monotonic decrease of the cost functional and numerical stability. The methodology is applied to compliance minimization, shape reconstruction, and inverse problems; the results demonstrate clear advantages in stability, convergence, and interpretability compared with classical level-set methods.

Chapter 5 is devoted to optimization in nonlinear elasticity. A discontinuous Galerkin scheme is introduced for level-set evolution, enabling robust simulations under complex mechanical conditions and supporting topology optimization in nonlinear settings.

The candidate then provides code listings for a three-dimensional frictional contact problem and for two-dimensional shape optimization.

The bibliography, comprising 67 references, spans a broad range of works thematically related to the dissertation.

3. Substantive evaluation

The dissertation addresses an important and timely research area of significant scientific and engineering relevance. It presents a comprehensive study of optimal control as well as shape and topology optimization in elastic systems, combining a coherent theoretical framework with efficient numerical methods.

The work covers a wide spectrum, including linear and nonlinear elasticity, contact problems, and control-based structural design, aiming to build a unified approach bridging mathematical analysis and computation.

The study begins by formulating an optimal boundary control problem in linear elasticity, where force–stress is treated as the control variable. The problem is transformed into state and adjoint equations together with optimality conditions by minimizing a cost functional that includes displacement tracking and control energy.

The dynamic control problem is approximated by its static counterpart through the use of the turnpike property, which indicates that the optimal trajectory tends toward a stationary solution over longer horizons, thereby substantially reducing computational costs.

In the context of shape and topology optimization, the dissertation employs shape derivatives together with velocity-field methods to guide structural evolution. Casting the problem as a gradient flow turns the optimization into a time-dependent system of partial differential equations, improving numerical stability and facilitating convergence analysis. The method proves highly effective for compliance optimization, shape reconstruction, and inverse problems, surpassing the classical level-set approach in interpretability, theoretical rigor, and algorithmic control.

For nonlinear elasticity, a discontinuous Galerkin method is used to solve the level-set evolution equations. This enables accurate and robust simulation of the optimization process, allowing effective treatment of demanding mechanical conditions. Numerical experiments confirm the practical utility of the proposed approach.

The contact problem is addressed via two approaches. The first uses phase-field regularization to model the contact interface continuously, enabling sensitivity analysis and smooth structural evolution. The second employs topological derivatives to assess the impact of interior perturbations away from the contact boundary, combined with the phase-field approach for optimization. Together, these provide a flexible and effective toolkit for shape and topology optimization under contact.

Shape optimization is also analyzed through a gradient-flow formulation for the static contact problem, using the elastic energy and Kohn–Vogelius-type functionals.

The main achievements of the dissertation include:

- the capability to treat both linear and nonlinear elastic systems;
- the use of regularization ensuring the existence of optimal solutions in shape and topology optimization problems;
- computational methods based on shape gradient flow, a relatively new area in shape optimization, for which convergence can be obtained at the continuous level;
- 2D and 3D computational results indicating broad applicability of the developed methodology;
- proprietary software for contact analysis and shape optimization.

In summary, the dissertation presents a coherent approach to the control and optimization of elastic systems, combining strong theoretical underpinnings with efficient numerical methods and demonstrating effectiveness in computational simulations. The results attest to the candidate's very good command of the research area, inventiveness, and professionalism. The structure of the dissertation is logical and well thought out.

Points for discussion

- (i) How complex and large would the problem be if the turnpike property were not exploited and the task were solved numerically as a fully dynamic problem?
- (ii) Does the proposed methodology for shape and topology optimization guarantee finding a

global optimum? If not, what possibilities does the candidate see for seeking a global solution?

(iii) What possibilities does the candidate see for employing selected AI methods to support the shape and topology optimization tasks formulated in the dissertation?

An important element of the dissertation, demonstrating the candidate's strong grasp of the subject matter, is the clear presentation of perspectives and plans for future research.

4. Final conclusion

The doctoral dissertation of M.Sc. Yixin Tan is a successful attempt at an integrated view of the control and optimization of elastic systems. It makes an important theoretical contribution to shape sensitivity analysis, gradient flow, and control mechanisms, and proposes effective numerical schemes. These results provide a foundation for future dynamic optimization, including coupling of multiple physical phenomena and further exploration of turnpike phenomena in elastic systems. The research results included in the dissertation testify to the candidate's extensive knowledge and experience.

Taking the foregoing assessment into account, I state that the doctoral dissertation by M.Sc. Yixin Tan fully meets the requirements for doctoral theses.

The candidate is well prepared to conduct independent scientific research.

Therefore, I consider that the submitted dissertation satisfies the conditions set for doctoral theses under the currently applicable law and I request that it be admitted to public defense.



Tadeusz Burczyński