

Abstract

With the rapid growth of problems tackled by information technology, the need for efficient algorithms keeps increasing. Choosing an effective algorithm for a specific problem (or a class of problems) usually requires both theoretical and numerical study, which, in turn, requires a corresponding toolset to assist the research process.

One important class of algorithms with wide application area are algorithms for solving variational inequalities, as it models many tasks in operations research, economics, traffic flow, game theory, and some problems of modern machine learning. Because these problems are large and complex, algorithmic efficiency is critical. To solve a problem with algorithms from the class we require both theory – applicability and convergence analysis under realistic assumptions – and computation-systematic experiments that reveal practical speed and robustness.

This dissertation provides numerical and theoretical research on iterative algorithms for variational inequalities together with a software framework, which allows solving corresponding class of problems and investigating behavior of algorithms.

Theoretical part devoted to obtaining new or improved algorithms for specific classes of problems and proving their convergence under widest possible assumptions.

The software framework simplifies experiments with new and modified algorithms and problems, because to add a new method or a problem a user can extend provided base classes and make only minimal changes. Also, for the implemented problems and algorithms it is possible to make different runs, set parameters from command line or code and obtain detailed execution log together with convergence graphs.

Most examples of practical applications are based on the problems of the so-called network economy, when some kind of optimization or equilibrium problem is considered on the network or graph. For the blood supply chain model a graphical UI is provided (web), which allows setting parameters for nodes and edges, selecting algorithms subset and obtaining the solution together with detailed execution logs and convergence graphs without modifying the python code.

The dissertation has classic structure. The first chapter is devoted to the literature review about modeling practical problems with variational inequalities and developing corresponding algorithms.

The second chapter describes a new variant of extragradient algorithm with monotone step adjustment together with proof of convergence under maximally wide assumptions.

The third chapter is devoted to a new version of extrapolation from the past algorithm, leveraging the idea of special distance (Bregman divergence) to simplify computations for a special class of problems. Corresponding proof of convergence also provided.

The fourth chapter is about constructing the analogue of the extrapolation from the past algorithm without requiring the problem space being Hilbert space, with corresponding convergence proof.

The fifth (the last) chapter is devoted to describing the software framework architecture and usage scenarios, together with demonstration of applicability of the described algorithms to the problem of blood supply chain network optimization, problem of traffic equilibrium search and PageRank problem. This chapter also demonstrates the full cycle from formulating a model for a problem and rewriting it as a variational inequality, to obtaining a solution and analyzing the behavior of algorithms on the selected problem using capabilities of the developed software system.