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Review of the Doctoral Thesis

Algorithmic information complex based on the variational
inequalities for optimization problems in networks

Author: Serhii DENYSOV

The monograph contains 90 pages, divided into 5 chapters, abstract, concluding remarks, `quoted references, and author scientific achievements. It is devoted to a class of algorithms which are used to solve variational inequalities. It contains new or modified algorithms applied in a variety of problems, proofs of their convergence and numerical investigations of their properties.

The author defines variational inequality and points out that variational inequalities are applied in a broad class of optimization problems. For instance in operation research, economics, transportation networks, game theory, machine learning. For some of them investigated algorithms are implemented.

The first chapter is devoted to the literature review. It contains mathematical models (based on the known results) of several optimization problems in which variational inequalities are used (see Subchapter 1.3). Then, several types of algorithms applied in these models are presented in Subchapter 1.4. For example economic algorithms such as extragradient algorithm and its modification in Hilbert space (Algorithm 1.1) and algorithm known as algorithm from the past are discussed in details. Moreover adaptive algorithms, algorithms with Non-Euclidean distance and in Hadamard spaces are also presented.

The second chapter describes extragradient algorithm with monotone step, together with the proof of its possible convergence. Moreover, algorithm for approximate solution of variational inequalities in Hilbert spaces is proposed and studied. Finally, algorithm for operator equation is proposed.

In Chapter 3 algorithms with accelerated projection are presented and discussed. First, using results taken directly from the theory of Hilbert spaces and Bregman Divergence, algorithms are described and their properties are discussed. Then two algorithms with proofs of their convergence are proposed.

At the end of Chapter 3 based on the use of Bregman divergence, adaptive variant of the extrapolation from the past algorithm is discussed. Moreover, different relationships with the previous results on variational inequalities are given.

Chapter 4 presents extrapolation methods for variational inequalities defined in uniformly convex Banach spaces. Most of the results are based on geometry of Banach spaces .

First, in Subchapter 4.2 Algorithm 4.1 is presented and then in Subchapter 4.3 certain additional inequalities are proposed (Theorem 4.1 and Corollary 4.1) and in Theorem 4.2 weak convergence of sequences generated by Algorithm 4.1 is proved.

Special type of adaptive extrapolation problem is formulated and solved in Subchapter 4.4, in which Algorithm 4.2 is proposed and its possible convergence is discussed in Lemma 4.6. Finally, it should be pointed out that the results of Chapter 4 are valid in special class of uniformly convex Banach spaces. Finally in Theorem 4.3 weak convergence of Algorithm 4.2 is proved using series of additional results given in Lemma 4.6.

Moreover, special two optimization problems in general linear spaces are formulated and considered. The first problem is solved using method in which standard Euclidean projection is replaced by Albert's generalized projection.

It should be mentioned that the second problem and its convergence proof are based on the first proposed algorithm. Theorems on weak convergence of the investigated algorithms are also proved. Finally, extrapolation is also discussed and its convergence are analyzed.

In the last Chapter 5, the software framework architecture for Hilbert space and applications of the algorithms in medicine and in the optimization problems are considered. Diagrams for the above mentioned problems are presented and some computational results are discussed.

Chapter 5 contains also results of numerical experiments together with their graphical interpretations. The relationships between variational inequalities and networks economic systems are discussed. PageRank problem is also formulated and solved using variational inequality methods. Finally, relationships between network economic problems and variational inequalities are considered.

The main results of the Dissertation are as follows:

1. Adaptive algorithm for solving variational inequalities on the set of fixed points of an auxiliary operator in Hilbert space and proof of its convergence.
2. Adaptive algorithm based on Bregman divergence has been proposed and theoretically justified.
3. A software framework allows combining modeled problem with the set of algorithms.
4. Theoretical results can be used for developments of more efficient approaches to solving problems modeled by variational inequalities.

Main author contributions have been published in international well known in the field journals (4 papers) and international conferences materials (7 papers). However, all author publications are with coauthors.

In the thesis, advanced mathematical tools are used in the proofs. For instance functional analysis methods are widely exploited.

Reference list is extremely reach – it contains 119 publications.

Summarizing, reviewing monograph presents solutions of certain optimization problems. The monograph is well organized and satisfies all demands necessary to obtain the PhD degree in computer science.

Jerry Klencik